SINGLE JET MIXING AT ARBITRARY ANGLE IN TURBULENT TUBE FLOW

Project F004

Report 4

to the

MEMBER COMPANIES OF THE INSTITUTE OF PAPER SCIENCE AND TECHNOLOGY

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By

Feng, Z., Wang, X., and Forney, L.J.

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Single Jet Mixing at Arbitrary Angle in Turbulent Tube Flow

by

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Abstract

An asymptotic procedure is presented to evaluate the tracer trajectory in a two-stream turbulent pipe mixing unit with an oblique branch. The proposed mixing jet trajectory estimate near the injection point is compared with the existing experimental data and used to calculate the critical mixing configurations. In addition, it is shown that the well-known tracer jet profile can be recovered for the case of a normally issued tracer turbulent jet.

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1 Introduction

The problem of mixing two fluid streams by turbulent jet injection into a pipeline has various applications in areas such as chemical reactions, heat transfer operations, and mixing and combustion processes in industry. As a simple but effective method to mix two fluids, pipeline mixers have been studied extensively. The first systematic study was conducted by Chilton and Genereaux [1], in which smoke visualization techniques were used to determine the optimum mixing conditions at a glass tee. Forney and Kwon [3] proposed a similarity law, which was derived from approximate solutions to the equations of motion for the case of a single, fully developed turbulent jet issuing normally to the flow. Forney and Lee [4] further established the importance of the diameter and velocity ratio for geometrically similar flows. Maruyama et al. [8] [9] studied the jet injection of fluid into the pipeline over several pipe diameters from the injection point, and they proposed the standard deviation as a mixing quality indicator. Ger and Holley [5] and Fitzgerald and Holley [2] conducted some experiments and compared standard deviations of measured tracer concentrations far downstream from the side tee. Sroka and Forney [10] derived a scaling law for the second moment of the tracer concentration within the pipeline when the turbulent jet injection is normal to the pipeline.

However, most of the research has been conducted for a configuration in which the jet is normal to the pipeline. In the present study, we consider a more general case in which the turbulence jet injects fluid at an angle θ_o ($0^o < \theta_o < 180^o$), and we derive asymptotic solutions for both jet trajectory and tracer concentration profiles in the near region of the jet injection point. The proposed analytical solutions are compared with the existing experimental results for turbulent mixing of two fluid streams at an oblique branch [8] [9] and the analytical solutions for T-junctions [3].

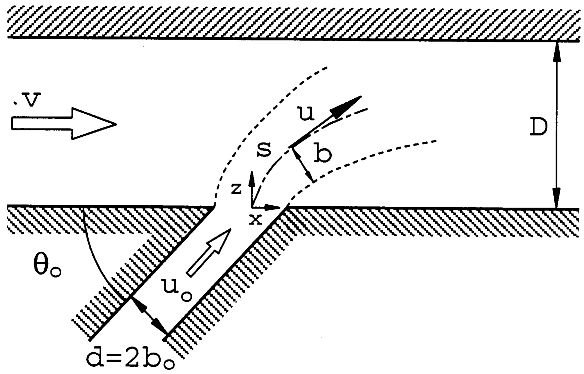

Figure 1: Two fluid streams mixing at an oblique branch.

In chemical engineering it is desirable to have the side-issued jet contact the opposite wall in order to enhance rapid mixing. However, in the paper industry, the tracer jet is often issued at an angle θ_o ($45^o \leq \theta_o \leq 60^o$) to avoid contact with the opposite wall and, in this way, to minimize flow disturbance and pressure pulsation. The presented analytical solution of the tracer jet trajectory will provide valuable information on the conditions under which the tracer jet will contact the opposite wall. In addition, such straightforward estimates of jet trajectories can be used to confirm both experimental and computational data.

2 Jet Injection at an Arbitrary Angle

2.1 Theory

The configuration of a general pipeline mixer with an angle θ_o is shown in Fig. 1, in which a jet with diameter d (or radius $b_o = d/2$) issues fluid containing tracer into a tube of diameter D. The ambient fluid velocity of the tube is v, and the initial tracer jet velocity is u_o. The phenomenon of jet mixing of a tracer in turbulent tube flow involves two phases. In the initial stage, the mixing process is dominated by self-induced jet turbulence. After a distance, the jet evolves into a geometrically centered jet, and the mixing of the tracer is dominated by turbulence in the main stream. Forney and Kwon [3] proposed a characteristic length l_m, which represents the distance over which the jet travels before it bends over in the cross flow. This momentum length is defined as follows:

$$l_m = \frac{du_o \sin\theta_o}{v}. \tag{1}$$

For convenience, we also introduce the following dimensionless length:

$$R = \frac{l_m/\sin\theta_o}{d} = \frac{u_o}{v}. \tag{2}$$

2.2 Field Equations

Our goal is to derive an asymptotic expression for the jet trajectory at the first stage, i.e., close to the jet orifice. The governing equations for the present problem include the conservation of mass and momentum. The well-known entrainment model first developed by Hoult, Fay, and Forney [6] is employed in this paper. The model assumes that there are two additive entrainment mechanisms, one is due to the tangential difference between the local jet velocity u and ambient fluid velocity component parallel to the jet, and the other to the ambient fluid velocity normal to the jet. For the configuration in Fig. 1, we can write the conservation of mass as follows:

$$\frac{1}{2b}\frac{d}{ds}(b^2 u) = \alpha(u - v\cos\theta) + \beta v \sin\theta, \tag{3}$$

where s, θ, u, b, α, and β stand for the mixing jet's arc length, tangential angle, jet velocity, equivalent cross-sectional radius, and the tangential and normal entrainment parameters, respectively.

The conservation of tangential momentum can be written as:

$$\frac{d}{ds}(b^2u^2) = v\cos\theta\frac{d}{ds}(b^2u), \tag{4}$$

and similarly for the normal momentum, we have

$$b^2u^2\frac{d\theta}{ds} = -v\sin\theta\frac{d}{ds}(b^2u). \tag{5}$$

The conservation of tracer concentration c gives

$$\frac{d}{ds}(cb^2u) = 0. \tag{6}$$

The boundary conditions at the jet orifice are specified as:

$$s = 0, \quad \theta = \theta_o, \quad u = u_o, \quad b = b_o, \quad c = c_o.$$

2.3 Asymptotic Solutions

Because we are interested in an asymptotic expression for the jet trajectory close to the orifice, we assume that the departure of θ from θ_o is small; furthermore, the Reynolds number of the orifice is restricted to large values to ensure jet turbulence, and the effect of buoyancy is neglected. Under these assumptions, from Eq. (3), we have

$$\frac{b^2u}{b_o^2u_o} \simeq 1 + 4\Omega\frac{s}{d}, \tag{7}$$

in which Ω is a constant given by

$$\Omega = \alpha(1 - \frac{\cos\theta_o}{R}) + \beta\frac{\sin\theta_o}{R}, \tag{8}$$

and from Eqs. (4) and (5), we obtain

$$\frac{b^2u^2}{b_o^2u_o^2} = \frac{\sin\theta_o}{\sin\theta}. \tag{9}$$

Furthermore, Eqs. (5) and (9) give us

$$R\frac{d\theta}{ds} = -\frac{\sin^2\theta}{\sin\theta_o}\frac{d}{ds}(\frac{ub^2}{u_ob_o^2}). \tag{10}$$

By integrating Eq. (10), we have, within the first-order approximation,

$$\theta \simeq \theta_o - 4\Omega\sin^2\theta_o\frac{s}{l_m}, \tag{11}$$

and consequently, by the nature of Eq. (6), we obtain

$$\frac{c}{c_o} = \frac{1}{1 + 4\Omega s/d}. \tag{12}$$

To convert the above expression into cartesian coordinates (x, z), we introduce the following relations:

$$dz = ds\sin\theta = ds\sin(\theta_o + \delta\theta) \simeq ds(\sin\theta_o + \cos\theta_o\delta\theta), \tag{13}$$

$$dx = ds\cos\theta = ds\cos(\theta_o + \delta\theta) \simeq ds(\cos\theta_o - \sin\theta_o\delta\theta). \tag{14}$$

For the region near the orifice, we have

$$\delta\theta = \theta - \theta_o \simeq -4\Omega\sin^2\theta_o\frac{s}{l_m}. \tag{15}$$

Integrating Eqs. (13) and (14), we obtain the following two key parametric equations for the asymptotic jet trajectory:

$$z = s\sin\theta_o - \Omega\sin\theta_o\sin 2\theta_o s^2/l_m, \quad (16)$$

$$x = s\cos\theta_o + 2\Omega\sin^3\theta_o s^2/l_m. \quad (17)$$

Furthermore, the tracer trajectory is given implicitly by the following relation:

$$x^2\cos^2\theta_o + xz\sin 2\theta_o + z^2\sin^2\theta_o - \frac{l_m}{2\Omega\sin^2\theta_o}(x\sin\theta_o - z\cos\theta_o) = 0, \quad (18)$$

while the tracer concentration profile, in cartesian coordinates, becomes

$$\frac{c}{c_o} = \frac{1}{1 + R(\sqrt{\cos^2\theta_o + 8\Omega\sin^3\theta_o x/l_m} - \cos\theta_o)/\sin^2\theta_o}. \quad (19)$$

In particular, for the case of normal jet injection, in which $\theta_o = 90^o$, Eq. (18) reduces to

$$\frac{z^2}{l_m^2} - \frac{x}{2\Omega l_m} = 0, \quad (20)$$

or

$$\frac{z}{l_m} = \sqrt{\frac{x}{2l_m\alpha + 2l_m\beta/R}} \quad (21)$$

which is exactly the same result as given by Forney and Kwon [3]. As pointed out by Forney and Kwon [3], although Eq. (21) is restricted to the condition $x/l_m \ll 1$, i.e., valid for the region close to the orifice, a numerical solution obtained by Hoult and Weil [7], indicates that the range of validity of Eq. (21) can be extended away from the orifice, and there are no significant deviations between the approximate result and the numerical result until $x \gg l_m$. For the general case of the present problem, further numerical computation will be needed to confirm the claim that Eq. (18) can be extended away from the near field region where $x/l_m \ll 1$.

2.4 Impact on the Opposite Wall

In chemical engineering, it is assumed that optimal mixing and reaction take place when the issued jet impacts the opposite wall, while in the paper industry, in order to minimize the pressure pulsation and flow disturbance in the approach flow system, it is desirable to avoid having the jet impact on the wall. Therefore, the following estimate of those conditions under which the jet trajectory will intercept the wall, i.e., the conditions under which there exists a solution of Eq. (16) yielding an x for $z = D$, plays an important role in the design of pipeline mixers.

From Eq. (16), by substituting $z = D$, we obtain the arc length s from the start point to the first intercept point with the opposite wall,

$$s = \frac{\sin\theta_o - \sqrt{\sin^2\theta_o - 4D\Omega\sin\theta_o\sin 2\theta_o/l_m}}{2\Omega\sin\theta_o\sin 2\theta_o/l_m}. \tag{22}$$

The corresponding intercept coordinate x_i is then calculated using Eq. (17). Of course, the existence of such a solution requires that

$$R\frac{d}{D}\sin^2\theta_o \geq 8\cos\theta_o\Omega, \tag{23}$$

where the equal sign yields the critical jet injection angle at which the impact on the opposite wall will happen.

From Eqs. (16) and (17), we also obtain

$$x\sin\theta_o - z\cos\theta_o = 2\Omega\sin^2\theta_o s^2/l_m. \tag{24}$$

Therefore, we may conclude that $x_i > D\text{ctan}\theta_o$, which is consistent with the fact that the nearest possible position at which the jet can impact the opposite tube wall is $D\text{ctan}\theta_o$, when the tube ambient fluid velocity $v = 0$.

2.5 Correlation of Data

The solutions of the two parametric equations for the asymptotic jet trajectory Eqs. (16) and (17) are plotted and compared with the experimental data of Maruyama et al. [8] [9]. The comparison with seven branch angles covering the range from $30^o \sim 150^o$ is shown in Fig. 2. The tangential and normal entrainment coefficients are chosen as $\alpha = 0.11$ and $\beta = 0.6$, which are the so-called universal constants discussed in Refs. [6] [7]. Moreover, Hoult and Weil [7] also concluded that the entrainment constants have constant values which for α is known to within 20 per cent accuracy and for β is to 25 per cent accuracy.

The experimental data represent hot-wire anemometer measurements of maximum jet velocities and, in this case, may not represent the geometric center of the jet, and they are the actually measured data reported in detail in Refs. [8] [9]. We find that the correlation is good for branch angles $\theta_o \leq 90^o$ while the asymptotic solutions deviate significantly from the measured data for $\theta_o > 90^o$. Our understanding is that in the latter case, the jet is projected upstream and turns abruptly near the origin, and the assumption of small deviation of θ from θ_o is violated. Nevertheless, the fact that in all branch angle cases the predicted trajectories match with the experimental measurements near the injection point confirms the assumption we use in deriving the parametric equations (16) and (17), which in fact explains the reason why we only present the asymptotic solutions near the injection point. Finally, we note that the real jet trajectory will bend down when it experiences the effects or existence of the opposite wall and the asymptotic estimate of the impact

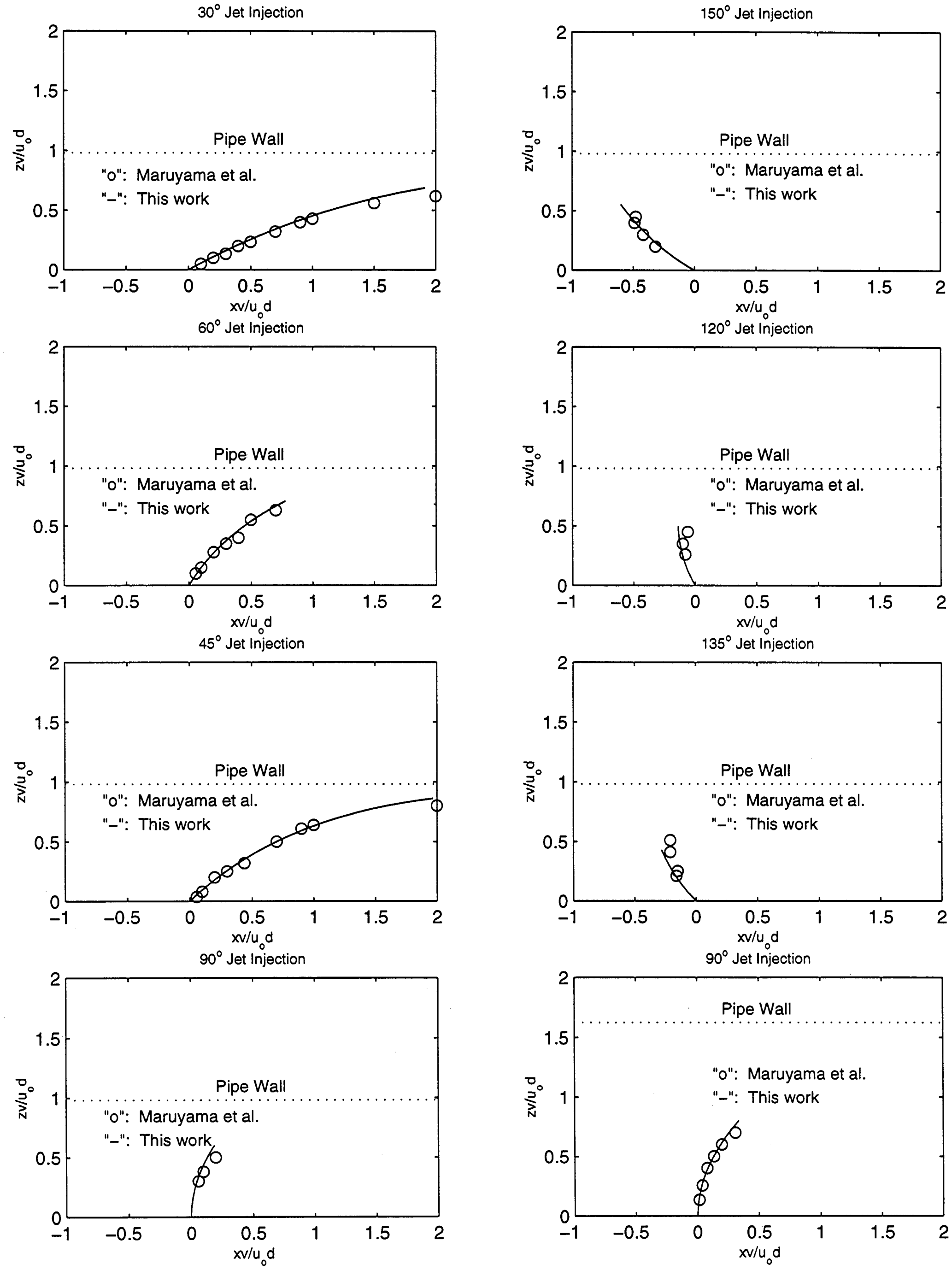

Figure 2: Mixing stream trajectories near the injection points at different angles. For the bottom-right case, $d = 0.8$ cm, $D = 5.1$ cm, $R = 3.9$; For the other cases, $d = 1.3$ cm, $D = 5.1$ cm, $R = 4.0$.

point is on the conservative side.

3 Conclusion

The problem of a turbulent jet in a crossflow at an arbitrary injection angle is studied analytically. By employing the entrainment model and exploring the conservation equations of mass, momentum, and tracer, we derived asymptotic solutions for jet trajectory and tracer concentration of a region close to the jet injection point under the assumptions of a near region where ambient turbulence on the mixing process can be neglected relative to jet-induced turbulence. The proposed asymptotic solutions straightforward and match well with the existing experimental data. In addition, a critical jet injection angle estimate is also presented.

4 Acknowledgment

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